Fun with Sum and Difference **My name is:_____**

#1-6. Using the sum & difference identities, condense each of the following and express as a trig function of a single angle.

1. $\sin 97^o \cos 43^o + \cos 97^o \sin 43^o$

2. $\cos 72^o \cos 130^o + \sin 72^o \sin 130^o$

3. $\dfrac{\tan 140^o - \tan 60^\circ}{1 + \tan 140^o \tan 60^\circ}$

4. $\sin\dfrac{\pi}{5}\cos\dfrac{2\pi}{3} - \cos\dfrac{\pi}{5}\sin\dfrac{2\pi}{3}$

5. $\cos\dfrac{\pi}{6}\cos\dfrac{\pi}{7} - \sin\dfrac{\pi}{6}\sin\dfrac{\pi}{7}$

6. $\dfrac{\tan\dfrac{\pi}{3} + \tan\dfrac{\pi}{4}}{1 - \tan\dfrac{\pi}{3}\tan\dfrac{\pi}{4}}$

#7-8. Use the sum & difference identities with unit circle values to find exact answers for the following:

7. cos(-105)

8. $\sin 345^o$

#9-11. Given: $\csc\alpha = \dfrac{13}{5}$, $\dfrac{\pi}{2} \le \alpha \le \pi$, and $\tan\beta = -\dfrac{3}{4}$, $\dfrac{3\pi}{2} \le \beta \le 2\pi$, find the following:

9. $\sin(\alpha - \beta)$

10. $\cos(\beta + \alpha)$

11. $\tan(\alpha - \beta)$

#12-13. If $\sin\theta = -\dfrac{3}{5}$ and θ is in the third quadrant, find the following:

12. $\cos(\theta + \dfrac{\pi}{3})$

13. $\tan 2\theta$

#14-18. Verify the following identities.

14. $\sin(\pi - x) = \sin x$

15. $\sin(\dfrac{3\pi}{2} + x) = -\cos x$

16. $\cos(30^0 - x) + \cos(30^o + x) = \sqrt{3}\cos x$

17. $\dfrac{\sin(\beta - \alpha)}{\sin\alpha \sin\beta} = \cot\alpha - \cot\beta$

18. $\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2\cos\alpha\cos\beta$

#19-21. Solve each of the following equations over the interval $[0, 2\pi)$.

19. $\sin\left(x + \dfrac{\pi}{6}\right) - \sin\left(x - \dfrac{\pi}{6}\right) = \dfrac{1}{2}$

20. $\tan(x + \pi) + 2\sin(x + \pi) = 0$

21. $\sin\left(x + \dfrac{\pi}{2}\right) - \cos\left(x + \dfrac{3\pi}{2}\right) = 0$

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1. $\sin 97\cos 43 + \cos 97\sin 43 =$

$\sin(97+43) = \boxed{\sin(140^\circ)}$

2. $\cos 72\cos 130 + \sin 72\sin 130$

$\cos(72-130) = \boxed{\cos(-58)}$

3. $\dfrac{\tan 140 - \tan 60}{1 + \tan 140\tan 60} = \tan(140-60)$

$= \boxed{\tan(80^\circ)}$

4. $\sin\frac{\pi}{5}\cos\frac{2\pi}{3} - \cos\frac{\pi}{5}\sin\frac{2\pi}{3}$

$\sin\left(\frac{\pi}{5} - \frac{2\pi}{3}\right) = \sin\left(\frac{3\pi}{15} - \frac{10\pi}{15}\right)$

$\boxed{\sin\left(\frac{-7\pi}{15}\right)}$

5. $\cos\frac{\pi}{6}\cos\frac{\pi}{7} - \sin\frac{\pi}{6}\sin\frac{\pi}{7}$

$= \cos\left(\frac{\pi}{6} + \frac{\pi}{7}\right) = \cos\left(\frac{7\pi}{42} + \frac{6\pi}{42}\right)$

$= \boxed{\cos\left(\frac{13\pi}{42}\right)}$

6. $\dfrac{\tan\frac{\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{3}\tan\frac{\pi}{4}} = \tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

$= \boxed{\tan\left(\frac{7\pi}{12}\right)}$

7. $\cos(-105) = \cos(45-150) =$

$\cos 45\cos 150 + \sin 45\sin 150$

$\frac{\sqrt{2}}{2}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)$

$= \boxed{\dfrac{-\sqrt{6}+\sqrt{2}}{4}}$

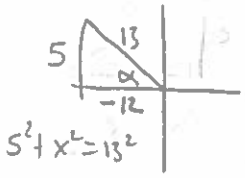
8. $\sin 345 = \sin(300+45)$

$= \sin 300\cos 45 + \cos 300\sin 45$

$\left(\frac{-\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \boxed{\dfrac{-\sqrt{6}+\sqrt{2}}{4}}$

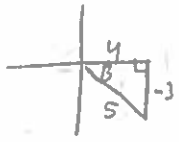
9-11 $\csc\alpha = \frac{13}{5}$ H, O



$5^2 + x^2 = 13^2$

$x = 12$

$\tan\beta = \frac{-3}{4}$



9. $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\left(\frac{5}{13}\right)\left(\frac{4}{5}\right) - \left(\frac{-12}{13}\right)\left(\frac{-3}{5}\right)$

$\frac{20}{65} - \frac{36}{65} = \frac{-16}{65}$

10. $\cos(\beta+\alpha)$

$= \cos\beta\cos\alpha - \sin\beta\sin\alpha$

$\left(\frac{4}{5}\right)\left(\frac{-12}{13}\right) - \left(\frac{-3}{5}\right)\left(\frac{5}{13}\right)$

$\frac{-48}{65} + \frac{15}{65} = \boxed{\frac{-33}{65}}$

11. $\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta} = \dfrac{-\frac{5}{12} - \frac{-3}{4}}{1 + \frac{-5}{12}\cdot\frac{-3}{4}}$

$\dfrac{-\frac{5}{12} + \frac{9}{12}}{1 + \frac{5}{16}} = \dfrac{\frac{4}{12}}{\frac{21}{16}} = \frac{4}{12}\cdot\frac{16}{21} = \boxed{\frac{16}{63}}$

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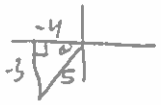
#12-13 $\sin\theta = -\frac{3}{5}$



12. $\cos\left(\theta + \frac{\pi}{3}\right)$

$\cos\theta\cos\frac{\pi}{3} - \sin\theta\sin\frac{\pi}{3}$

$\left(\frac{-4}{5}\right)\left(\frac{1}{2}\right) - \left(\frac{-3}{5}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\frac{-4}{10} + \frac{3\sqrt{3}}{10} = \boxed{\frac{-4+3\sqrt{3}}{10}}$

13. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2}$

$\frac{\frac{3}{2}}{1-\frac{9}{16}} = \frac{\frac{3}{2}}{\frac{7}{16}} = \frac{3}{\cancel{2}_1} \cdot \frac{\overset{8}{\cancel{16}}}{7} = \boxed{\frac{24}{7}}$

14. $\sin(\pi - x) = \sin x$

$\sin\pi\cos x - \cos\pi\sin x$

$0\cdot\cos x - (-1)\sin x$

$\boxed{\sin x = \sin x}$

15. $\sin\left(\frac{3\pi}{2} + x\right) = -\cos x$

$\sin\frac{3\pi}{2}\cos x + \cos\frac{3\pi}{2}\sin x$

$(-1)\cos x + 0\cdot\sin x$

$\boxed{-\cos x = -\cos x}$

16. $\cos(30 - x) + \cos(30 + x) = \sqrt{3}\cos x$

$\cos 30\cos x + \sin 30\sin x + \cos 30\cos x - \sin 30\sin x$

$\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$

$2\frac{\sqrt{3}}{2}\cos x$

$\boxed{\sqrt{3}\cos x = \sqrt{3}\cos x}$

17. $\frac{\sin(\beta - \alpha)}{\sin\alpha\sin\beta} = \cot\alpha - \cot\beta$

$\frac{\sin\beta\cos\alpha - \cos\beta\sin\alpha}{\sin\alpha\sin\beta}$

$\frac{\cancel{\sin\beta}\cos\alpha}{\sin\alpha\cancel{\sin\beta}} - \frac{\cos\beta\cancel{\sin\alpha}}{\cancel{\sin\alpha}\sin\beta}$

$\frac{\cos\alpha}{\sin\alpha} - \frac{\cos\beta}{\sin\beta}$

$\boxed{\cot\alpha - \cot\beta = \cot\alpha - \cot\beta}$

18. $\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2\cos\alpha\cos\beta$

$\cos\alpha\cos\beta - \sin\alpha\cancel{\sin\beta} + \cos\alpha\cos\beta + \cancel{\sin\alpha}\sin\beta$

$\boxed{2\cos\alpha\cos\beta = 2\cos\alpha\cos\beta}$

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19. $\sin\left(x+\frac{\pi}{6}\right)-\sin\left(x-\frac{\pi}{6}\right)=\frac{1}{2}$

$\sin x\cos\frac{\pi}{6}+\cos x\sin\frac{\pi}{6}-\left[\sin x\cos\frac{\pi}{6}-\cos x\sin\frac{\pi}{6}\right]=\frac{1}{2}$

$\sin x\cdot\frac{\sqrt{3}}{2}+\cos x\cdot\frac{1}{2}-\sin x\cdot\frac{\sqrt{3}}{2}+\cos x\cdot\frac{1}{2}=\frac{1}{2}$

$\cos x=\frac{1}{2}$

$x=\frac{\pi}{3},\frac{5\pi}{3}$

20. $\tan(x+\pi)+2\sin(x+\pi)=0$

$\frac{\tan x+\tan\pi}{1-\tan x\tan\pi}+2\sin x\cos\pi+2\cos x\sin\pi=0$ ($\tan\pi=0$)

$\tan x+-2\sin x=0$

$+2\sin x \quad +2\sin x$

$\tan x=2\sin x$

$\frac{\sin x}{\cos x}=2\sin x$

$\frac{\sin x}{\sin x}=\frac{2\sin x\cos x}{\sin x}$

$\sin x=0$

$x=0,\pi$

$1=2\cos x$

$\cos x=\frac{1}{2}$

$x=\frac{\pi}{3},\frac{5\pi}{3}$

21. $\sin\left(x+\frac{\pi}{2}\right)-\cos\left(x+\frac{3\pi}{2}\right)=0$

$\sin x\cos\frac{\pi}{2}+\cos x\sin\frac{\pi}{2}-\left(\cos x\cos\frac{3\pi}{2}-\sin x\sin\frac{3\pi}{2}\right)=0$

($\cos\frac{\pi}{2}=0$, $\cos\frac{3\pi}{2}=0$, $\sin\frac{3\pi}{2}=-1$)

$\cos x-\sin x=0$

$\cos x=\sin x$

$x=\frac{\pi}{4},\frac{5\pi}{4}$